Title: The Calibration Principle of the Cern Hall Probes to the 10^{-4} accuracy for the Muon to Electron Experiment

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Abstract:

The Mu2e experiment requires a high precision map of the magnetic field with an accuracy of 10^{-4} Tesla in order to obtain the momentum distribution of decay particles and understand locations of potential magnetic traps that could contribute to the number of background events. The precision 3D Hall probes used in this study were only precise to a 10^{-3} Tesla within a limited temperature range, and thus they need to be re-calibrated to achieve better accuracy.

The purpose of this study was to develop tools to create a multidimensional fit to interpolate 3D Hall probe readings between obtained calibration data points. The factors evaluated include probe orientation, ambient temperature, and current magnetic field. Three interpolation methods were explored in this study: 1) a linear spline of the average B-field, 2) a linear spline of the B-field components, and 3) a linear-sinusoidal fit of the B-field components. The average B-field linear spline method was accurate to 10^{-5} T but could only be applicable within a homogeneous magnetic field. The B-field component linear spline method was accurate to 10^{-4} T, but the individual components were only accurate to $10-^{3}$ T. Therefore, these two methods would not be suitable for the Mu2e experiment. The linear-sinusoidal method was accurate to 10^{-4} T for both the average B-field and each B-field component and is the best method.

Introduction:

The Muon to Electron (Mu2e) experiment is a project of Fermilab to search for evidence of new physics beyond the Standard Model in the form of charged lepton flavor violation by looking for neutrino-less muon to electron conversion in the field of the nucleus [1].

Mu2e uses several superconducting magnets to create a complex magnetic field to transport muons from their production point to the decay target, where several detectors will be used to analyze the decay products [2]. To correctly identify the particle tracks, a high precision map of the magnetic field within the experiment is required to obtain the momentum distribution of decay particles and understand locations of potential magnetic traps that could contribute to the number of background events. In Mu2e, 3D Hall probes will be used to create the field map, which demands a precision of 10^{-4} Tesla [3]. Currently, the industry standard for the accuracy of magnetic field probes is 10^{-3} Tesla within a limited temperature range [4]. Thus, these probes need to be re-calibrated. There are many challenges to calibrating 3D Hall probes to an accuracy of 10^{-4} Tesla, such as probe temperature, magnet temperature and current [5][6][7], the planar and 3D Hall effects [8][9], and the time needed to measure probe orientation and Hall voltage more accurately at smaller intervals [4]. In this study, we explored the possibility of calibrating 3D Hall probes to 10^{-4} Tesla.

Experiment:

Equipment

A GMW 3474-250mm Calibration Magnet was used to create the homogenous, well-known magnetic field used in calibration. The magnet was powered by a DanFysik 8500 series power supply equipped with a Keithley 2002 + 2001 DMM for current and voltage readout. The magnet was cooled by a custom industrial cooling water/low conductivity water heat exchanger and pump. Two types of magnetic field probes were used: a PT2026 NMR Precision Teslameter and Bergsma design CERN Hall probes with BatCan readout. The Hall probes were mounted on custom Hall probe mounts which were attached onto a SmarAct SR-7021 rotary positioner. Both probes were positioned using a Zaber X-LRT-DE series three-axis linear stage. The entire setup was enclosed by a temperature enclosure, using a T224 HVAC system to keep the temperature constant. The data from each instrument were collected using software designed in the LabVIEW 2016 Development Kit and analyzed using MATLAB 2018.

Methods

The NMR teslameter was used to map the magnetic field of the calibration magnet to determine the stable, homogeneous portion of the field, as well as its field strength. A total of 400 data points were collected at this portion of the field using a 3D Hall probe, with 20 rotation angles in both θ and ϕ directions. At each data point, the average magnetic field (B-field) strength and the x, y, and z components of the B-field (B_x, B_y, and B_z, respectively) were measured. Sets of data at two θ angles were randomly taken out and used as validation sets. The remaining 360-point dataset was used for data analysis. Data for B_x, B_y, and B_z of this dataset were plotted against the θ angle.

Several approaches were explored to create an interpolation of the magnetic field data. First, a simple linear spline interpolation method was used to create a piecewise linear fit of the average magnetic field data (ie.) composed of line segments between data points. The second approach used was a linear spline interpolation each of B_x, B_y, and B_z. The final approach used was a sinusoidal fit of the B_y and B_z data and a linear fit of the B_x data (linear-sinusoidal method). After each interpolation attempt, interpolated data was compared with the validation datasets.

Results and Discussion:

The results obtained from the B-field linear spline interpolation method are shown in Table 1. The average accuracy from this method is 1.67×10^{-5} T. As shown in Table 2, the linear spline interpolation by B-field component method had an average B-field accuracy of 1.84×10^{-5} T. The Bx, By, and Bz components had average accuracies of 5.99×10^{-3}, 1.39×10^{-3}, and 1.14×10^{-3} T respectively. The results obtained from the linear and sinusoidal fits by B-field component are summarized in Table 3 and the average B-field accuracy is 3.45×10^{-5} T. The B_x, B_y, and B_z components had average accuracies of 2.03×10^{-5}, 3.03×10^{-5}, and 5.62×10^{-5} T respectively.

Although the linear spline interpolation method had the highest B-field accuracy, the B-field itself was extremely homogeneous, with a range of 3.78×10^{-5} T as shown in Table 4. Thus, this method would not be suitable for non-homogenous or complex fields, such as that of the Mu2e experiment. The linear spline interpolation by B-field component method is accurate to 10^{-5} T for the average B-field, but is not accurate for each individual component. Therefore, this method would not be suitable for the Mu2e experiment. The linear-sinusoidal method had an accuracy less than 10^{-4} T for both the average B-field and each B-field component, and thus would be the the best method for the calibration of Hall probes in the Mu2e experiment.

Table 1: Linear spline interpolation of average B-field data

φ (rad)	Average B-field (T)	B-field difference (T)
-4.363E-02	1.4490283	4.69E-06
-3.904E-02	1.4490201	-1.62E-05
-3.445E-02	1.4490088	-7.53E-06
-2.985E-02	1.4490054	3.37E-05
-2.526E-02	1.4489985	2.93E-05
-2.067E-02	1.4489898	1.80E-05
-1.608E-02	1.4490026	-3.52E-05
-1.148E-02	1.4490045	6.65E-06
-6.889E-03	1.4490170	9.15E-07
-2.296E-03	1.4490080	1.85E-05
2.297E-03	1.4490246	7.34E-06
6.889E-03	1.4490240	1.58E-05
1.148E-02	1.4490265	-1.70E-05
1.608E-02	1.4490194	-8.91E-06
2.067E-02	1.4490184	-2.32E-05
2.526E-02	1.4490198	-1.95E-05
2.985E-02	1.4490142	5.56E-06
3.445E-02	1.4490227	6.47E-06
3.904E-02	1.4490182	-3.16E-05
4.363E-02	1.4490273	-2.80E-05

Table 2: Linear spline interpolation of B_x, B_y, and B_z data

φ (rad)	B_x (T)	B_x difference (T)	B_y (rad)	B_y difference (T)	B_z (T)	B_z difference (T)	Average B-field (T)	B-field difference (T)
-4.363E-02	-0.0022483	-2.99E-05	0.8816447	-2.65E-03	1.1499239	2.06E-03	1.4490091	2.39E-05
-3.904E-02	0.0044084	-6.62E-03	0.8829973	1.31E-03	1.1488873	-1.01E-03	1.4490154	-1.16E-05
-3.445E-02	0.0044177	-6.67E-03	0.8882600	1.28E-03	1.1448022	-9.84E-04	1.4489988	2.55E-06
-2.985E-02	0.0044262	-6.64E-03	0.8935076	1.29E-03	1.1407066	-9.50E-04	1.4489951	4.40E-05
-2.526E-02	0.0044415	-6.62E-03	0.8987194	1.32E-03	1.1365994	-9.94E-04	1.4489908	3.70E-05
-2.067E-02	0.0044668	-6.66E-03	0.9039385	1.29E-03	1.1324415	-9.87E-04	1.4489819	2.60E-05
-1.608E-02	0.0044847	-6.66E-03	0.9091258	1.23E-03	1.1282889	-1.01E-03	1.4489878	-2.04E-05
-1.148E-02	0.0045038	-6.67E-03	0.9142926	1.27E-03	1.1241075	-1.00E-03	1.4489889	2.22E-05
-6.889E-03	0.0045267	-6.65E-03	0.9194440	1.24E-03	1.1199275	-1.01E-03	1.4490118	6.06E-06
-2.296E-03	0.0045318	-6.63E-03	0.9245587	1.26E-03	1.1156847	-9.92E-04	1.4489934	3.31E-05
2.297E-03	0.0045449	-6.66E-03	0.9296757	1.25E-03	1.1114504	-1.02E-03	1.4490134	1.86E-05
6.889E-03	0.0045618	-6.70E-03	0.9347572	1.25E-03	1.1071764	-1.01E-03	1.4490104	2.94E-05
1.148E-02	0.0045797	-6.71E-03	0.9398273	1.24E-03	1.1028879	-1.06E-03	1.4490197	-1.01E-05
1.608E-02	0.0045962	-6.67E-03	0.9448716	1.27E-03	1.0985551	-1.08E-03	1.4490089	1.64E-06
2.067E-02	0.0045967	-6.66E-03	0.9499120	1.21E-03	1.0941986	-1.06E-03	1.4490081	-1.29E-05
2.526E-02	0.0046058	-6.65E-03	0.9549215	1.21E-03	1.0898311	-1.07E-03	1.4490094	-9.02E-06
2.985E-02	0.0046126	-6.65E-03	0.9599017	1.26E-03	1.0854404	-1.08E-03	1.4490043	1.55E-05
3.445E-02	0.0046153	-6.64E-03	0.9648818	1.23E-03	1.0810267	-1.07E-03	1.4490124	1.69E-05
3.904E-02	0.0046411	-6.65E-03	0.9698335	1.21E-03	1.0765750	-1.10E-03	1.4490039	-1.73E-05
4.363E-02	-0.0020210	3.52E-05	0.9735223	2.43E-03	1.0732543	-2.22E-03	1.4490081	-8.85E-06

Table 3: Linear fit of B_x data and sinusoidal fit of B_y and B_z data

φ (rad)	Bx (T)	Bx difference (T)	By (rad)	By difference (T)	Bz (T)	Bz difference (T)	Average B-field (T)	B-field difference (T)
-4.363E-02	-0.0022649	-1.32E-05	0.8790652	-7.03E-05	1.1518674	1.13E-04	1.4489855	4.74E-05
-3.904E-02	-0.0022460	3.27E-05	0.8843249	-2.12E-05	1.1478248	4.90E-05	1.4489780	2.58E-05
-3.445E-02	-0.0022400	-1.25E-05	0.8896141	-7.55E-05	1.1437456	7.29E-05	1.4489901	1.12E-05
-2.985E-02	-0.0022422	3.15E-05	0.8948712	-7.25E-05	1.1396215	1.35E-04	1.4489778	6.13E-05
-2.526E-02	-0.0022243	4.09E-05	0.9000685	-2.55E-05	1.1355233	8.21E-05	1.4489794	4.84E-05
-2.067E-02	-0.0021925	-1.57E-07	0.9052619	-3.85E-05	1.1313881	6.64E-05	1.4489800	2.79E-05
-1.608E-02	-0.0021992	2.42E-05	0.9104041	-4.62E-05	1.1272547	2.09E-05	1.4489802	-1.28E-05
-1.148E-02	-0.0021677	5.95E-06	0.9156143	-4.90E-05	1.1230245	8.23E-05	1.4489784	3.28E-05
-6.889E-03	-0.0021513	3.12E-05	0.9206904	-4.74E-06	1.1188558	6.61E-05	1.4489699	4.80E-05
-2.296E-03	-0.0021430	4.01E-05	0.9258054	9.46E-06	1.1146350	5.80E-05	1.4489759	5.06E-05
2.297E-03	-0.0021292	1.12E-05	0.9308646	6.55E-05	1.1104047	2.69E-05	1.4489693	6.27E-05
6.889E-03	-0.0021174	-1.61E-05	0.9360095	-4.66E-07	1.1060976	6.64E-05	1.4489894	5.04E-05
1.148E-02	-0.0020953	-3.87E-05	0.9410670	1.68E-06	1.1017990	2.40E-05	1.4489901	1.94E-05
1.608E-02	-0.0020957	1.73E-05	0.9461179	1.89E-05	1.0974471	2.84E-05	1.4489767	3.38E-05
2.067E-02	-0.0020691	7.37E-06	0.9511474	-2.36E-05	1.0931033	3.28E-05	1.4489860	9.21E-06
2.526E-02	-0.0020610	1.76E-05	0.9561488	-1.48E-05	1.0887117	5.16E-05	1.4489713	2.90E-05
2.985E-02	-0.0020463	1.04E-05	0.9611418	1.61E-05	1.0843175	3.94E-05	1.4489797	4.01E-05
3.445E-02	-0.0020194	-8.16E-06	0.9660875	2.41E-05	1.0798945	6.38E-05	1.4489655	6.37E-05
3.904E-02	-0.0020355	2.99E-05	0.9710491	-7.27E-06	1.0754643	5.70E-06	1.4489872	-6.86E-07
4.363E-02	-0.0020033	1.75E-05	0.9759754	-2.13E-05	1.0709932	3.84E-05	1.4489852	1.40E-05

Table 4: Validation dataset used

φ (rad)	Bx (T)	By (T)	Bz (T)	B (T)
-4.363E-02	-0.0022781	0.8789949	1.1519806	1.4490330
-3.904E-02	-0.0022133	0.8843037	1.1478738	1.4490038
-3.445E-02	-0.0022525	0.8895386	1.1438185	1.4490013
-2.985E-02	-0.0022106	0.8947987	1.1397564	1.4490391
-2.526E-02	-0.0021835	0.9000430	1.1356054	1.4490278
-2.067E-02	-0.0021926	0.9052235	1.1314545	1.4490078
-1.608E-02	-0.0021750	0.9103579	1.1272755	1.4489674
-1.148E-02	-0.0021617	0.9155653	1.1231068	1.4490112
-6.889E-03	-0.0021201	0.9206857	1.1189219	1.4490179
-2.296E-03	-0.0021029	0.9258148	1.1146929	1.4490265
2.297E-03	-0.0021180	0.9309301	1.1104316	1.4490320
6.889E-03	-0.0021335	0.9360090	1.1061640	1.4490398
1.148E-02	-0.0021341	0.9410686	1.1018230	1.4490096
1.608E-02	-0.0020784	0.9461367	1.0974755	1.4490105
2.067E-02	-0.0020618	0.9511238	1.0931361	1.4489952
2.526E-02	-0.0020434	0.9561340	1.0887633	1.4490004
2.985E-02	-0.0020359	0.9611578	1.0843569	1.4490198
3.445E-02	-0.0020275	0.9661116	1.0799583	1.4490292
3.904E-02	-0.0020055	0.9710418	1.0754700	1.4489865
4.363E-02	-0.0019858	0.9759540	1.0710316	1.4489993

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Works Cited:

[1] Fermilab | Mu2e | Research Goals. (n.d.). Retrieved May 19, 2019, from https://mu2e.fnal.gov/research_goals.shtml

[2] Fermilab | Mu2e | How Does It Work. (n.d.). Retrieved May 19, 2019, from https://mu2e.fnal.gov/how_does_it_work.shtml

[3] Buehler, M., Lopes, M. L.., Tartaglia, M., Tompkins, J., Gluchko, S., Orozco, C. (2011). *Mu2e Magnetic Measurements*. http://lss.fnal.gov/archive/2013/conf/fermilab-conf-13-268-td.pdf

[4] Calibration: the real price of Hall accuracy. (2010, November 20). Retrieved May 19, 2019, from Metrolab website: https://www.metrolab.com/technology-calibration-the-real-price-of-hall-accuracy/

[5] Restuccia, F. (2016). Calibration of a magnet used to calibrate Hall probes in Mu2e experiment [PowerPoint slides]. Retrieved from https://eddata.fnal.gov/lasso/summerstudents/papers/2016/Francesco-Restuccia.pdf

[6] Marchetti, D. (2016). *Calibration of a magnet used in Hall probe calibration in Mu2e experiment (Summer Internship 2016)*.

[7] Musardo, M., Corwin, T., Harder, D., He, P., Kitegi, C. A., Licciardi, W., … , Tanabe, T. (2013). 3D HALL PROBE CALIBRATION SYSTEM AT INSERTION DEVICE MAGNETIC MEASUREMENT FACILITY AT BNL, Proceedings of NA-PAC2013, Pasadena, CA, 2013.

[8] Sanfilippo, S. (2011). *Hall probes: physics and application to magnetometry*. Retrieved from https://arxiv.org/abs/1103.1271v1

[9] Bergsma, F. (2009). Developments in field mapping at CERN [PowerPoint slides]